Quiz.2 – Solution

Fall 2014-2015

(November 26, 2014)

CIVE210 – STATICS

CLOSED BOOK, 1 HR 30 Minutes

Name: Fall 2014 - 2015 **ID#:** 2015**** **Section:** 1,2,3,4,5,6,7

NOTES

- 2 PROBLEMS – 11 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- TWO EXTRA SHEETS IS PROVIDED AT THE END.
- ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- *DO NOT* USE THE BACK OF THE SHEETS FOR ANSWERS.
- *DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.*
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE TO CONFIRM THAT YOU HAVE SOLVED A QUESTION. ☑

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I: 35 /35

Problem II: 65 /65

TOTAL: 100 /100

Problem I: *(35 points)*

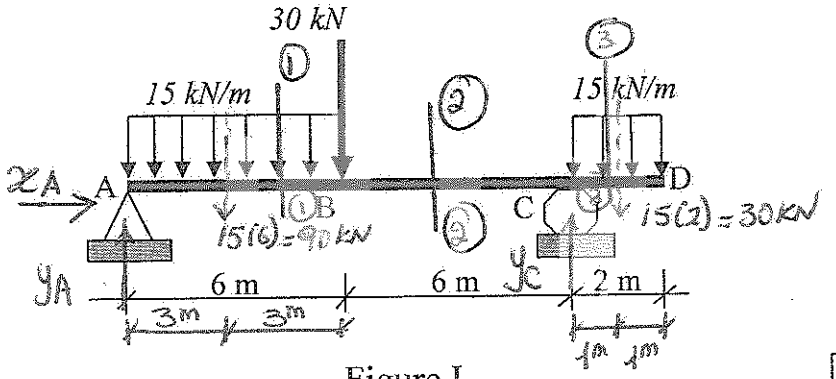
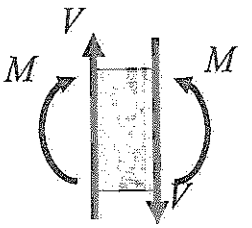


Figure I



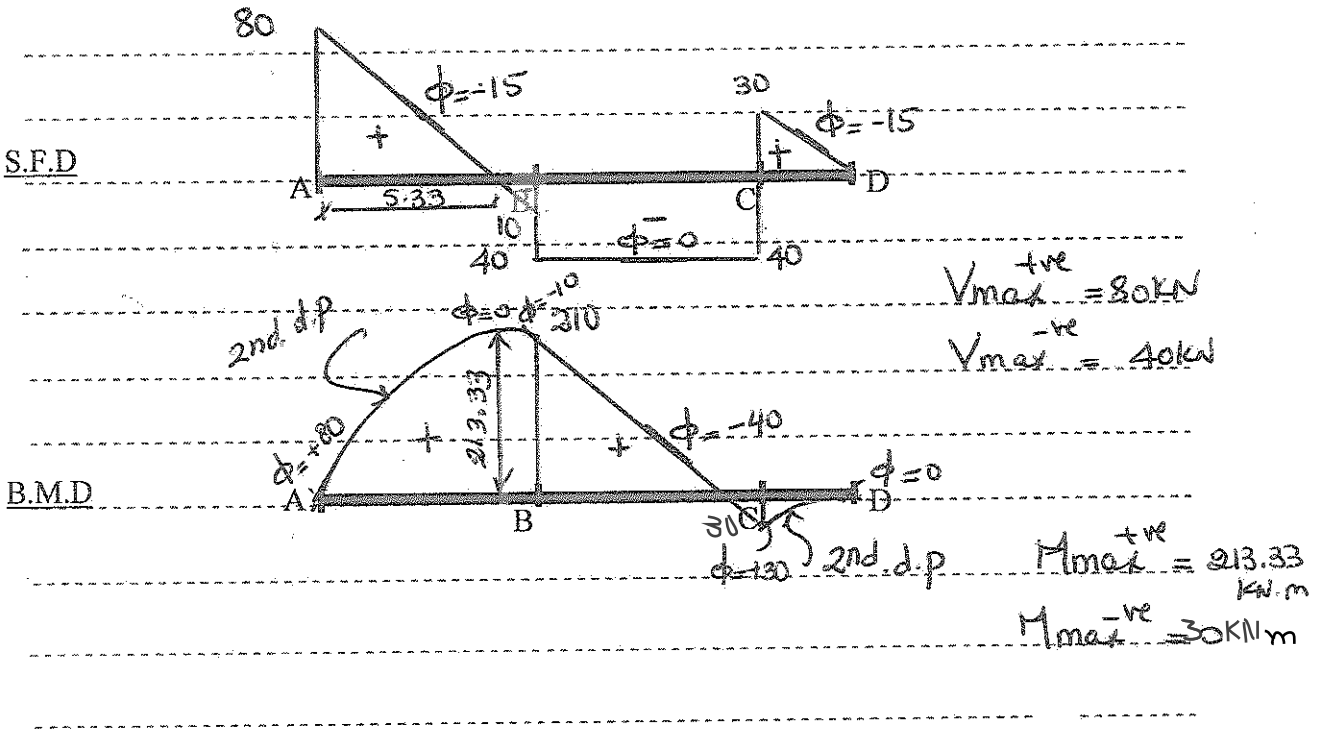
(+) Convention

Tick Boxes to check that you solved all questions

For the beam shown in Figure I:

1- Compute the reactions at supports A and C. *(5 points)* ☑
2- Using the method of sections, write the equations for shear and moments between A and B, B and C, and C and D. *(18 points)* ☑
3- Draw the shear force and bending moment diagrams (use the space provided below for the diagrams and draw to scale as much as you can). Show the important and necessary features and values on the diagrams and indicate the maximum positive and negative shears and moments in the beam. *(12 points)* ☑

Calculations and/or Diagrams:



Calculations and/or Diagrams (cont'd):

1- Reactions:

$\pm \rightarrow \sum F_x = 0 \Rightarrow x_A = 0$

$+ \circlearrowleft \sum M_A = 0 \Rightarrow -90(3) - 30(6) + y_C(12) - 30(13) = 0$

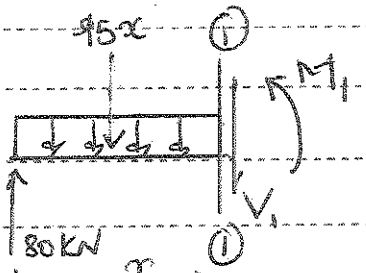
$\Rightarrow \boxed{y_C = 70\ KN \uparrow}$

$+\uparrow \sum F_y = 0 \Rightarrow y_A - 90 - 30 + 70 - 30 = 0$

$\Rightarrow \boxed{y_A = 80\ KN \uparrow}$

2- Sec ① - ① Left Part of the beam

$0 \leq x \leq 6m$ (A) (B)



$+\uparrow \sum F_y = 0 \Rightarrow 80 - 15x - V_1 = 0$

$\Rightarrow \boxed{V_1 = 80 - 15x}$ linear relation

$\begin{cases} \text{at } x = 0 \Rightarrow V_A = 80\ kN \text{ (Pin Support)} \\ \text{at } x = 6\ m \Rightarrow V_{B_L} = 80 - 15(6) = -10\ kN \end{cases}$

$+\circlearrowleft \sum M_{①} = 0 \Rightarrow M_1 + 15x\left(\frac{x}{2}\right) - 80x = 0$

$\Rightarrow \boxed{M_1 = 80x - 7.5x^2}$ 2nd d.p

$\begin{cases} \text{at } x = 0 \Rightarrow M_A = 0 \text{ (Pin Support)} \\ \text{at } x = 6m \Rightarrow M_B = 210\ KN.m \end{cases}$

Concavity: $\frac{dM}{dx} = 80 - 15x$; $\frac{d^2M}{dx^2} = -15$ ∩ all the way.

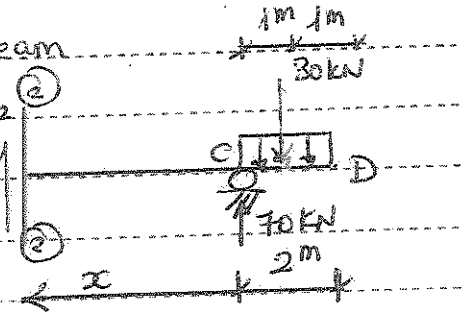
Zero shear location and M_{max}:

$V = 80 - 15x = 0 \Rightarrow x = \frac{80}{15} = 5.33\ m$

$M_{max} = 80(5.33) - 7.5(5.33)^2 = 213.33\ KN.m$

Calculations and/or Diagrams (cont'd):

Sec ② – ② ... right Part of the beam



$0 \le x \le 6m$ (C to B)

$+\uparrow \sum F_y = 0 \Rightarrow V_2 + 70 - 30 = 0$

$\boxed{V_2 = -40}$ constant

$\begin{cases} \text{at } x = 0\text{ m} \quad V_{C_L} = -40\text{ kN} \\ \text{at } x = 6\text{m} \quad V_{B_R} = -40\text{ kN} \end{cases}$

$\Delta V_B = V_{B_R} - V_{B_L} = -40 - (-10) = -30\text{ kN}$ ↙

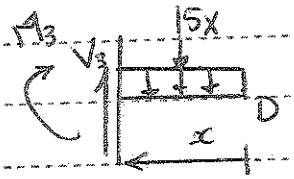
$+\circlearrowleft \sum M_{@②} = 0 \Rightarrow -M_2 + 70x - 30(x+1) = 0$

$\Rightarrow \boxed{M_2 = 40x - 30}$ linear relation

$\begin{cases} \text{at } x = 0 \quad M_{C_L} = -30\text{ kN.m} \\ \text{at } x = 6^m \quad M_B = 40(6) - 30 = 210\text{ kN.m} \quad \text{Same as Sec. ① – ①} \end{cases}$

Sec ③ – ③ Right Part of the beam

$0 \le x \le 2^m$ (D to C)



$+\uparrow \sum F_y = 0 \Rightarrow V_3 - 15x = 0 \Rightarrow \boxed{V_3 = 15x}$

linear relation

$\begin{cases} \text{at } x = 0 \Rightarrow V_D = 0 \\ \text{at } x = 2^m \Rightarrow V_{C_R} = 15(2) = 30\text{ kN} \end{cases}$

$\Delta V_C = V_{C_R} - V_{C_L} = 30 - (-40) = 70\text{ kN}$ ↙

$+\circlearrowleft \sum M_{@③} = 0 \Rightarrow -M_3 - 15x\left(\frac{x}{2}\right) = 0 \Rightarrow \boxed{M_3 = -7.5x^2}$ linear 2nd d.p

$\begin{cases} \text{at } x = 0 \Rightarrow M_D = 0 \text{ (free end)} \\ \text{at } x = 2^m \Rightarrow M_{C_R} = -7.5(2)^2 = -30\text{ kN.m} \quad \text{Same as Sec. ② – ②} \end{cases}$

EXTRA SHEET 1: Continued from page ____

Name: _____ **ID#:** _____

Calculations and/or Diagrams:

Concavity:

$\frac{dM}{dx} = -15x$; $\frac{d^2M}{dx^2} = -15$ All the way.

Problem II: *(65 points)*

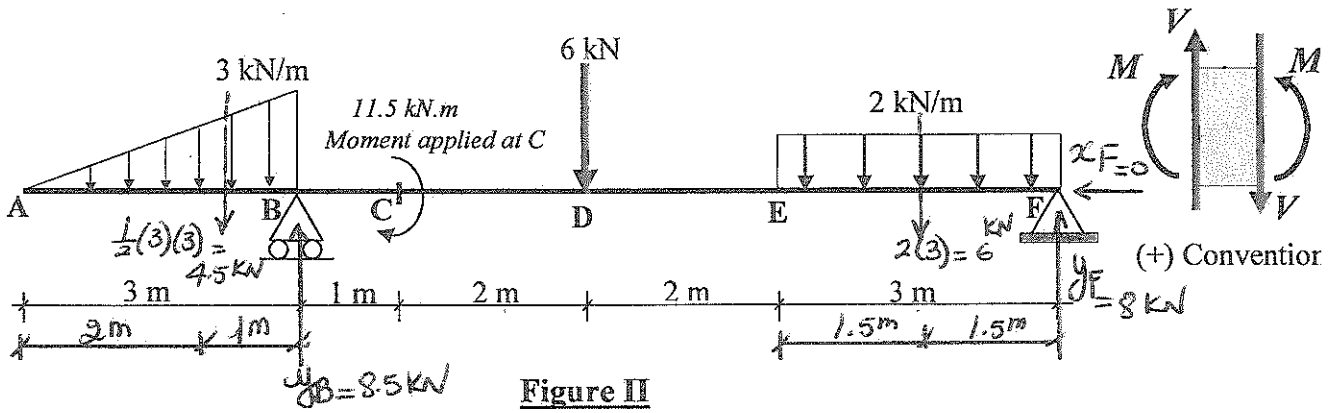


Figure II

For the beam shown in Figure II:

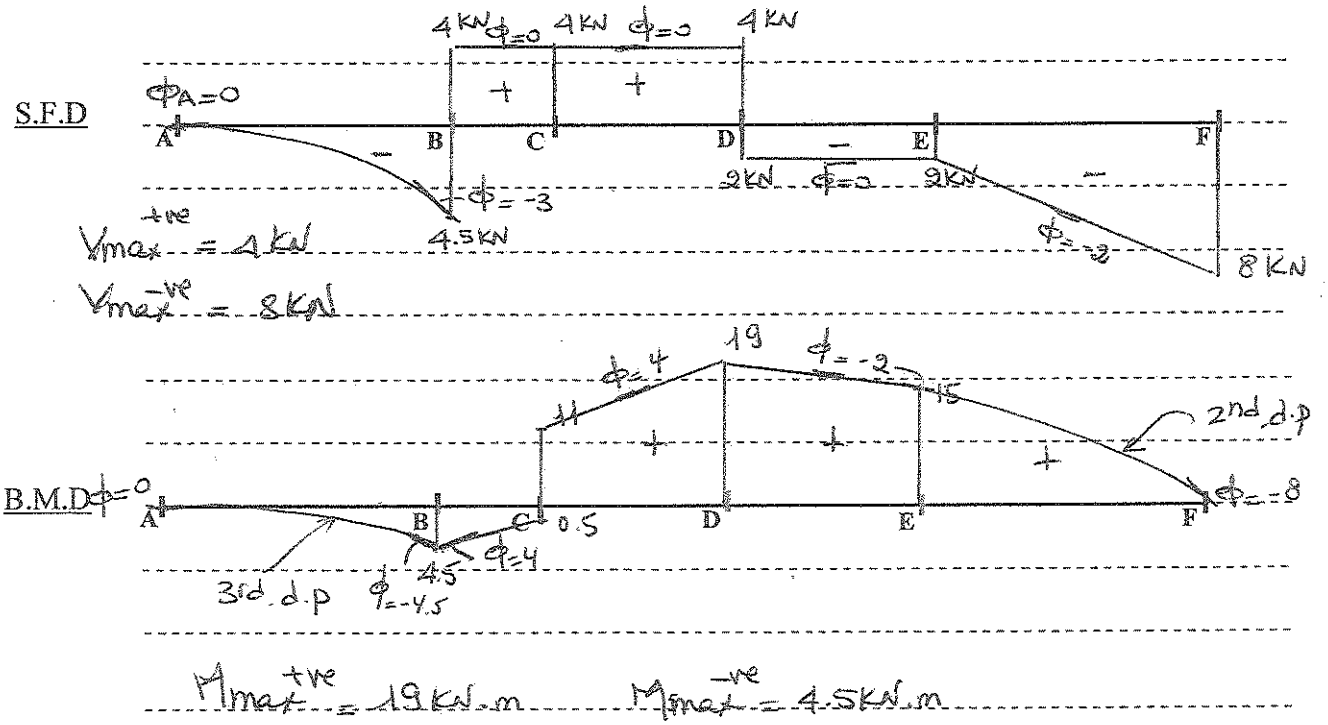
1- Compute the reactions at supports B and F. *(5 points)*

2- Using sections, compute the shear force and bending moments at points A, B,C, D,E, and F. *(20 points)*

3- Using a proper origin, write the equations for shear and moments between A and B and E and F, confirm your results obtained in question 2. *(10 points)*

4- Using the method of integration (or areas), draw the shear force and bending moment diagrams (use the space provided below for the diagrams and draw to scale as much as you can). Show the important and necessary features and values on the diagrams and indicate the maximum positive and negative shears and moments in the beam. *(30 points)*

Calculations and/or Diagrams:



$M_{max}^{+ve} = 19\ kN.m$ $M_{max}^{-ve} = 4.5\ kN.m$

Calculations and/or Diagrams (cont'd):

1- Reactions: $\pm\rightarrow \Sigma F_x = 0 \Rightarrow x_F = 0$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow 4.5(1) - 11.5 - 6(3) - 6(6.5) + y_F(8) = 0$

$\Rightarrow \boxed{y_F = 8.0\ KN \uparrow}$

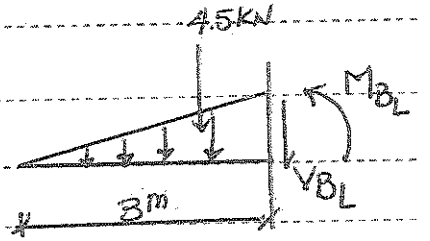
$+\circlearrowleft \Sigma M_F = 0 \Rightarrow 6(1.5) + 6(5) - y_B(8) + 4.5(9) - 11.5 = 0$

$\Rightarrow \boxed{y_B = 8.5\ KN \uparrow}$

check! $+\uparrow \Sigma F_y = 0 \Rightarrow -4.5 + 8.5 - 6 - 6 + 8.0 = 0$ ok

2- $V_A = 0$, $M_A = 0$ free end

at B_L:

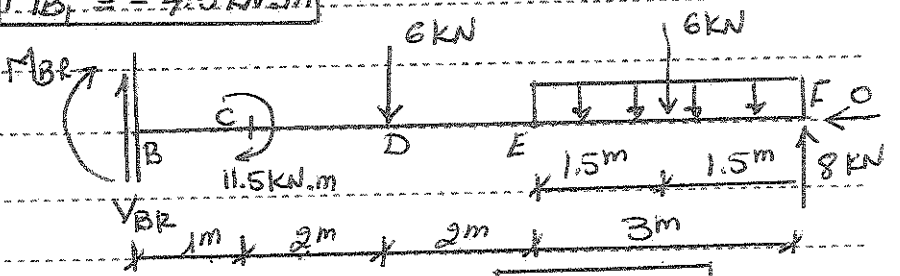


$+\uparrow \Sigma F_y = 0 \Rightarrow -4.5 - V_{B_L} = 0$

$\Rightarrow \boxed{V_{B_L} = -4.5\ KN}$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow M_{B_L} + 4.5(1) = 0$

$\Rightarrow \boxed{M_{B_L} = -4.5\ KN.m}$



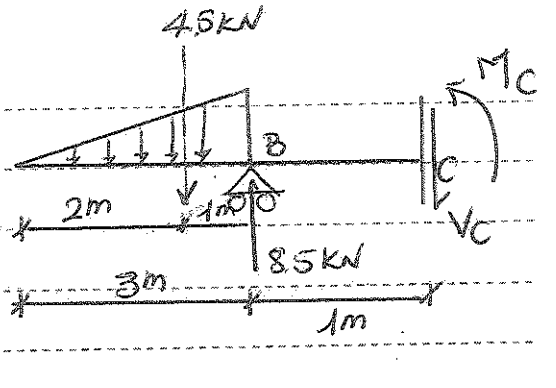
$+\uparrow \Sigma F_y = 0 \Rightarrow V_{BR} - 6 - 6 + 8 = 0 \Rightarrow \boxed{V_{BR} = 4\ KN}$

$\Delta V = V_{BR} - V_{BL} = 4 - (-4.5) = 8.5\ KN$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow -M_{BR} - 11.5 - 6(3) - 6(6.5) + 8(8) = 0$

$\Rightarrow \boxed{M_{BR} = -4.5\ KN.m}$ same as M_{BL}

Calculations and/or Diagrams (cont'd):



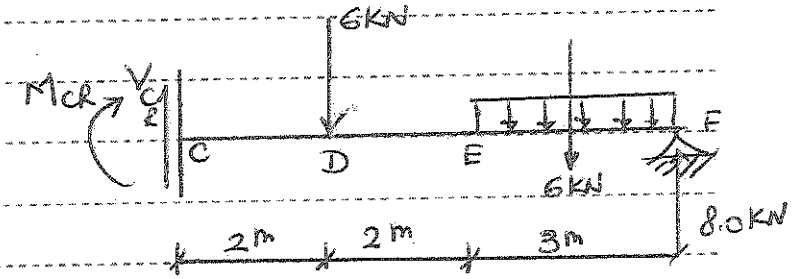
at C

$+\uparrow \sum F_y = 0 \Rightarrow -4.5 + 8.5 - V_C = 0$

$\Rightarrow \boxed{V_C = 4\ kN}$

$+\circlearrowleft \sum M_C = 0 \Rightarrow M_C - 8.5(1) + 4.5(2) = 0$

$\Rightarrow \boxed{M_{C_L} = -0.5\ kN.m}$



$+\uparrow \sum F_y = 0$

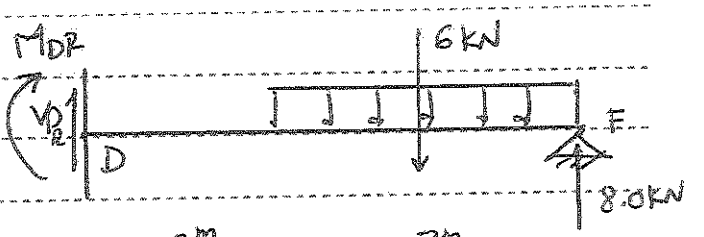
$\Rightarrow V_{CR} - 6 - 6 + 8 = 0$

$\Rightarrow V_{C_R} = 4\ kN$ ✓

$+\circlearrowleft \sum M_C = 0 \Rightarrow -M_{CR} - 6(2) - 6(5.5) + 8(7) = 0$

$\Rightarrow \boxed{M_{CR} = 11\ kN.m}$

$\Delta M = M_{CR} - M_C = 11 - (-0.5) = 11.5\ kN.m$ ok ✓



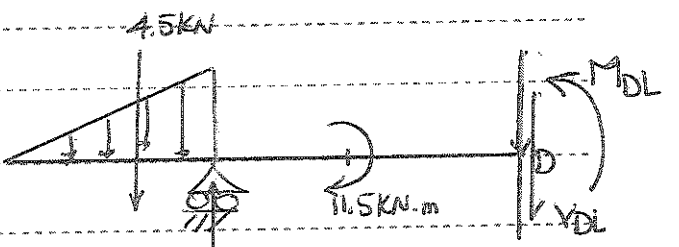
at D

$+\uparrow \sum F_y = 0 \Rightarrow V_{DR} - 6 + 8 = 0$

$\Rightarrow \boxed{V_{DR} = -2\ kN}$

$+\circlearrowleft \sum M_D = 0 \Rightarrow -M_{DR} - 6(3.5) + 8(5) = 0$

$\Rightarrow \boxed{M_{DR} = 19\ kN.m}$



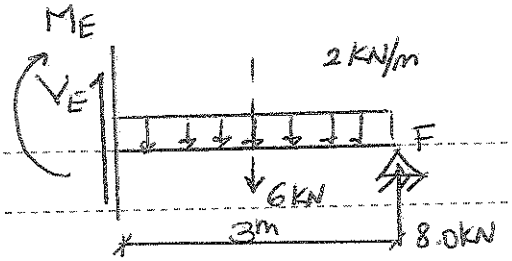
$+\uparrow \sum F_y = 0 \Rightarrow -4.5 + 8.5 - V_{DL} = 0$

$\Rightarrow \boxed{V_{DL} = 4\ kN}$

$+\circlearrowleft \sum M_D = 0 \Rightarrow 4.5(4) - 8.5(3) - 11.5 + M_{DL} = 0$

$\Rightarrow \boxed{M_{DL} = 19\ kN.m}$ ✓

Calculations and/or Diagrams (cont'd):



at E

$+\uparrow \sum F_y = 0 \Rightarrow V_E - 6 + 8 = 0$

$\Rightarrow \boxed{V_E = -2\,KN}$

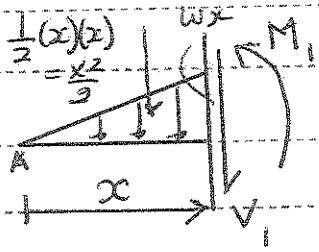
$+\circlearrowleft \sum M_E = 0 \Rightarrow -M_E - 6(1.5) + 8(3) = 0$

$\Rightarrow \boxed{M_E = 15\,KN.m}$

at F:

$V_F = +8\,KN \quad M_F = 0$

3- Sec ①-① left Part $0 \le x \le 3^m$ (A) (B)



By similar triangles: $\frac{3}{3} = \frac{w_x}{x} \Rightarrow w_x = x$

$+\uparrow \sum F_y = 0 \Rightarrow -\frac{x^2}{3} - V_1 = 0$

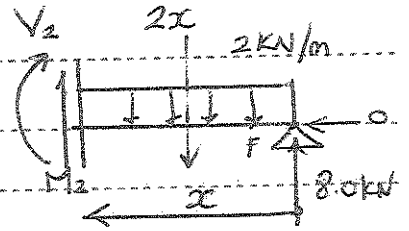
$\Rightarrow \boxed{V_1 = \frac{-x^2}{3}}$ (2nd order relation)

- at $x = 0 \Rightarrow V_A = 0$ (free end)
- at $x = 3^m \Rightarrow V_{B_L} = -\frac{(3)^2}{3} = -4.5\,KN$

$+\circlearrowleft \sum M_{@①} = 0 \Rightarrow M_1 + \frac{x^2}{3}\left(\frac{x}{3}\right) = 0 \Rightarrow \boxed{M_1 = \frac{-x^3}{6}}$

- at $x = 0 \Rightarrow M_A = 0$ (free end)
- at $x = 3^m \Rightarrow M_B = -\frac{(3)^3}{6} = -4.5\,KN.m$

Rectangular load: $0 \le x \le 3$ (E) (F)



$+\uparrow \sum F_y = 0 \Rightarrow V_2 - 2x + 8 = 0$

$\boxed{V_2 = 2x - 8}$

- at $x = 0 \Rightarrow V_F = 2(0) - 8 = -8\,KN$
- at $x = 3^m \Rightarrow V_E = 2(3) - 8 = -2\,KN$

Calculations and/or Diagrams (cont'd):

$+\circlearrowleft \sum M_{@2} = 0 \Rightarrow -M_2 - 2x\left(\frac{x}{2}\right) + 8x = 0$

$\Rightarrow \boxed{M_2 = 8x - x^2}$

$\begin{cases} \text{at } x = 0 \Rightarrow M_F = 0 \text{ (Pin Support)} \\ \text{at } x = 3^m \Rightarrow M_E = 8(3) - (3)^2 = 15\ KN.m \end{cases}$

Same as Q2

4-

$V_A = 0$

$V_{B_L} - V_A = -\frac{1}{2}(3)(3) = -4.5\ KN \Rightarrow V_{BL} = -4.5 KN$

$V_{BR} = V_{BL} + y_B = -4.5 + 8.5 = 4\ KN$

$V_C - V_{BR} = 0 \Rightarrow V_C = 4\ KN$

$V_{DL} - V_C = 0 \Rightarrow V_{DL} = 4\ KN$

$V_{DR} = V_{DL} - 6 = 4 - 6 = -2\ KN$

$V_E - V_{DR} = 0 \Rightarrow V_E = -2\ KN$

$V_F - V_E = -6 \Rightarrow V_F = -2 - 6 = -8\ KN = y_F$

$M_A = 0$

$M_B - M_A = -\frac{1}{3}(4.5)(3) = -4.5 \Rightarrow M_B = -4.5 KN.m$

$M_{C_L} - M_B = +4 \times 1 = 4 \Rightarrow M_{CL} = -4.5 + 4 = -0.5 KN.m$

$M_{CR} = M_{CL} + 11.5 = -0.5 + 11.5 = 11.0\ KN.m$

$M_D - M_{CR} = 4 \times 2 = 8 \Rightarrow M_D = 11.0 + 8 = 19\ KN.m$

$M_E - M_D = -2 \times 2 = -4 \Rightarrow M_E = 19 - 4 = 15 KN.m$

$M_F - M_E = -\left(\frac{2+8}{2}\right) \times 3 = -15 \Rightarrow M_F = 15 - 15 = 0$ ✓

EXTRA SHEET 2: Continued from page _____

Name: _____ **ID#:** _____

Calculations and/or Diagrams: